Int J Theor Phys (2009) 48: 2036–2040
DOI 10.1007/s10773-009-9979-z

Bianchi Type-V Inflationary Universe in General Relativity

D.R.K. Reddy

Received: 20 December 2008 / Accepted: 2 March 2009 / Published online: 12 March 2009

Abstract A Bianchi type-V space-time is considered in the presence of massless scalar field with a flat potential. To get an inflationary universe, we have considered a flat region in which potential V is constant. Some physical and kinematical properties of the model are also discussed.

Keywords Bianchi-V model · Inflationary universe · General relativity

1 Introduction

The study of early stages of evolution of the universe and the investigation of cosmological models do have astrophysical importance since they play a vital role in the structure formation of the galaxies. In particular, inflationary models of the universe are important in solving several outstanding problems in cosmology like homogeneity, the isotropy and the flatness of the observed universe. Guth [1], Linde [2] and La and Steinhardt [3] are some of the authors who have investigated different aspects of inflationary universes in general relativity.

In recent years there has been a lot of interest in the solutions of the Einstein equations where the scalar field is minimally coupled to the gravitational field. In particular, self-interacting scalar fields play a central role in the study of inflationary cosmology. Burd and Barrow [4], Wald [5], Barrow [6], Ellis and Madsen [7] and Heusler [8] studied many aspects of scalar fields in the evolution of the universe and FRW models. Bhattacharjee and Baruah [9], Bali and Jain [10] and Rahaman et al. [11] have studied the role of self-interacting scalar fields in inflationary cosmology.

Reddy et al. [12] and Reddy and Naidu [13] have discussed inflationary universes in general relativity in four and five dimensions. Recently, Reddy et al. [14] have investigated a plane symmetric Bianchi type-I inflationary universe in general relativity. In this paper, we

D.R.K. Reddy (✉)
Department of Science and Humanities, Maharaj Vijaya ram Gajapathi Raj College of Engineering, Vizianagaram, India
e-mail: reddy_einstein@yahoo.com

have presented a spatially homogeneous Bianchi type-V inflationary cosmological model in the presence of massless scalar field with a flat potential in general relativity. It is wellknown, at the present state of evolution, the universe, on the whole, is spherically symmetric and isotropic. But in the early stages of evolution, it could not have had such smoothed out picture. Therefore it is worthwhile to investigate spatially homogeneous and anisotropic cosmological models which provide a richer structure both geometrically and physically than FRW models and play significant role in the description and structure formation of the early universe. Thus, this study has significant astrophysical interest.

2 Metric and Field Equations

We consider the homogeneous and anisotropic Bianchi type-V space-time described by the line element

$$ds^2 = dt^2 - \exp(2A)dx^2 - \exp(2B + 2\mu x)dy^2 - \exp(2C + 2\mu x)dz^2 \tag{1}$$

where the metric potentials A, B and C are functions of cosmological time t and μ is the parameter.

In the case of gravity minimally coupled to a scalar field $V(\phi)$ the Lagrangian is

$$L = \int \left[R - \frac{1}{2}g^{ij}\phi_{,i}\phi_{,j} - V(\phi)\right]\sqrt{-g}d^4x \tag{2}$$

which on variation of L with respect to dynamical fields leads to Einstein field equations

$$G_{ij} = R_{ij} - \frac{1}{2}g_{ij}R = -T_{ij} \tag{3}$$

with

$$T_{ij} = \phi_{,i}\phi_{,j} - \left[\frac{1}{2}\phi_{,k}\phi^{,k} + V(\phi)\right]g_{ij} \tag{4}$$

$$\phi^{i}_{;i} = -\frac{dV}{d\phi} \tag{5}$$

where comma and semicolon indicate ordinary and covariant differentiation respectively. Other symbols have their usual meaning and units are taken so that

$$8\pi G = c = 1$$

Now the Einstein field equations (3) for the metric (1) can be written as

$$B_{44} + C_{44} + B_4^2 + C_4^2 + B_4C_4 - \mu^2 e^{-2A} + \frac{\phi_4^2}{2} + V(\phi) = 0 \tag{6}$$

$$A_{44} + C_{44} + C_4^2 + A_4^2 + C_4A_4 - \mu^2 e^{-2A} + \frac{\phi_4^2}{2} + V(\phi) = 0 \tag{7}$$

$$A_{44} + B_{44} + A_4^2 + B_4^2 + A_4B_4 - \mu^2 e^{-2A} + \frac{\phi_4^2}{2} + V(\phi) = 0 \tag{8}$$

$$A_4B_4 + B_4C_4 + C_4A_4 - 3\mu^2 e^{-2A} + \frac{\phi_4^2}{2} + V(\phi) = 0 \tag{9}$$

$$2A_4 - B_4 - C_4 = 0 \tag{10}$$

and (5) for the scalar field takes the form

$$\phi_{44} + \phi_4(A_4 + B_4 + C_4) = -\frac{dV}{d\phi} \tag{11}$$

Here the subscript 4 denotes differentiation with respect to t.

3 Inflationary Model

Here we are interested in inflationary solutions of the field equations (6)–(11).

Steinshabes [15] has shown that Higgs field ϕ with potential $V(\phi)$ has a flat region and the field evolves slowly but the universe expands in an exponential way due to vacuum field energy. It is assumed that the scalar field will take sufficient time to cross the flat region so that the universe expands sufficiently to become homogeneous and isotropic on the scale of the order of the horizon size. We consider the flat region where the potential is constant [15], i.e.

$$V(\phi) = \text{const.} = V_0(\text{say}) \tag{12}$$

Also, the field equations being highly non linear, to get a determinate solution we assume

$$B = mC \quad m \neq 1 \tag{13}$$

Now with the help of (12) and (13) the field equations (6)–(11) yield an exact solution given by

$$\begin{aligned} A &= \text{In}(t_0 t - t_1)^{\frac{m+1}{3(m-1)}} \\ B &= \text{In}(t_0 t - t_1)^{\frac{2m}{3(m-1)}} \\ C &= \text{In}(t_0 t - t_1)^{\frac{2}{3(m-1)}} \end{aligned} \tag{14}$$

$$\phi = m_0 \frac{(1-m)}{2}(t_0 t - t_1)^{\frac{-2}{m-1}} + \phi_0 \tag{15}$$

where m, m_0, t_0, t_1 and ϕ_0 are arbitrary constants.

After a suitable choice of coordinates and constants, Bianchi type-V inflationary model corresponding to the solution (14) takes the form

$$ds^2 = dT^2 - T^{\frac{2(m+1)}{3(m-1)}} dX^2 - e^{2\mu x}\left[T^{\frac{4m}{3(m-1)}} dY^2 + T^{\frac{4}{3(m-1)}} dZ^2\right] \tag{16}$$

and the scalar field ϕ becomes

$$\phi = m_0 \frac{(m-1)}{2} T^{\frac{-2}{m-1}} \tag{17}$$

4 Some Physical Properties of the Model

The model (16) represents an anisotropic inflationary universe in general relativity when the scalar field is minimally coupled to the gravitational field. The model has no initial singularity, i.e. at $T=0$. This model is similar to the Bianchi type-V string cosmological model discussed by Singh [16].

The physical and kinematical parameters for the model (16) have the following expressions:

$$\text{Spatial volume:}\quad V^3 = T^{\frac{m+1}{m-1}} \exp(2\mu x) \tag{18}$$

$$\text{Expansion scalar:}\quad \Theta = \frac{m+1}{(m-1)T} \tag{19}$$

$$\text{Shear scalar:}\quad \sigma^2 = \frac{(3m^2+2m+3)}{9(m-1)^2T^2} \tag{20}$$

$$\text{Hubble parameter:}\quad H = \frac{(m+1)}{3(m-1)T} \tag{21}$$

$$\text{Deceleration parameter:}\quad q = \frac{2(m-2)}{(m+1)} \tag{22}$$

$$\frac{\sigma^2}{\Theta} = \frac{(3m^2+2m+3)}{9(m^2-1)T} \tag{23}$$

The spatial volume increases with time T when $m+1>0$ and it becomes infinite for large T. Thus inflation is possible for large T. Also, the deceleration parameter q is constant. It can be observed that for large T the parameters Θ, σ, H vanish and they diverge for $m=1$ or when T approaches zero. The Higg's field ϕ diverges for $T=0$ and for large T it becomes zero. The ratio $\frac{\sigma^2}{\Theta}$ of the model tends to zero for large T which shows that the model approaches isotropy since no material source of anisotropy is present.

5 Conclusions

Spatially homogeneous and anisotropic Bianchi models play a vital role in understanding of the early stages of evolution of the universe. This study will throw some light on the structure formation of the universe which has astrophysical significance. Here we have studied Bianchi type-V anisotropic, homogeneous and inflationary cosmological model in the presence of mass less scalar field with a flat potential. The model obtained is non-singular, expanding and approaches isotropy at late times. This investigation important in view of the recent interest in the classical scalar fields in general relativity and in alternative theories of gravitation.

References

1. Guth, A.H.: Phys. Rev. D **23**, 347 (1981)
2. Linde, A.D.: Phys. Lett. B **108**, 389 (1982)
3. La, D., Steinhardt, P.J.: Phys. Rev. Lett. **62**, 2376 (1989)

4. Burd, A.B., Barrow, J.D.: Nucl. Phys. B **308**, 923 (1988)
5. Wald, R.: Phys. Rev. D **28**, 2118 (1983)
6. Barrow, J.D.: Phys. Lett. B **187**, 12 (1987)
7. Ellis, G.F.R., Madsen, M.S.: Class. Quantum Gravity **8**, 667 (1991)
8. Heusler, M.: Phys. Lett. B **253**, 33 (1991)
9. Bhattacharjee, R., Baruah, K.K.: Ind. J. Pure Appl. Math. **32**, 47 (2001)
10. Bali, R., Jain, V.C.: Pramana J. Phys. **59**, 1 (2002)
11. Rahaman, F., Bag, G., Bhui, B.C., Das, S.: Fizika B **12**, 193 (2003)
12. Reddy, D.R.K., Naidu, R.L., Rao, A.S.: Int. J. Theor. Phys. **47**, 1016 (2008)
13. Reddy, D.R.K., Naidu, R.L.: Int. J. Theor. Phys. **47**, 2339 (2008)
14. Reddy, D.R.K., Rao, A.S., Naidu, R.L., Astrophys. Space. Sci. (2007). DOI:10.1007/s10509-008-9955-8
15. Stein-Schabes, J.A.: Phys. Rev. D **35**, 2345 (1987)
16. Singh, J.K.: Int. J. Theor. Phys. (2007). DOI:10.1007/s10773-008-9863-2